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Repulsive force by vector mesons and quark-hadron phase transition

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Abstract

Using a phenomenological model with vector-type interactions, we discuss a role of repulsive force in the quark-hadron phase transition at high density. For realization of the quark phase at high density, strong vector coupling is needed in the hadron phase, while it is forbidden in the quark phase. For the quark-phase, a NJL-type model with a multi-quark interaction is investigated. We show that, in this model, the restoration of chiral symmetry decreases effective vector coupling and the quark phase is realized at high density, even if we have strong vector coupling at low density. In this model, the strong coupling in the hadron phase is induced by the chiral symmetry breaking.

1 Introduction

To study the QCD phase transition at high density, we consider several effective models, since lattice QCD is still not feasible due to the sign problem. For the confinement-deconfinement (hadron-quark) transition, we often use a so-called two-phase description in which the quark and the hadron phases are described separately. We compare two phases by the Gibbs criteria at the same temperature (T) and the same chemical potential, and it is concluded that a phase which has larger pressure is realized. It is well-known that repulsive force between nucleons is necessary for realization of the quark-phase at high density. [1] In a free gas model, the ratio of the pressure ($P_{\rm Q}$) of the quark phase to that ($P_{\rm H}$) of the hadron phase becomes $\frac{1}{27}$ in the high density limit, due to the relation $\mu = 3\mu_{\rm Q}$, where μ and $\mu_{\rm Q}$ are the chemical potential for the baryon and the quark numbers, respectively. Therefore, in a free gas model, the hadron phase is realized at high density. The repulsive force between nucleons is needed for the realization of the quark-phase. In this brief report, we investigate a role of vector-type repulsive interactions in the quark-hadron phase transition at high density.

2 Formalism

For hadron phase, we use the Walecka model [2] described by the following Lagrangian.

$$
\begin{aligned}
\mathcal{L}_{\rm WM} &= \bar{\psi}\left[\gamma^\mu\{i\partial_\mu + g_\omega\omega_\mu\} - \{M - g_\sigma\sigma\}\right]\psi \\
&+ \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu; \quad F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu. \quad (1)
\end{aligned}
$$

The ψ, σ, ω_μ, M, m_σ, m_ω, g_σ and g_ω are the nucleon field, the σ-meson field, the ω-meson field, the nucleon mass, the σ-meson mass, the ω-meson mass, the σ-nucleon coupling and the ω-nucleon coupling, respectively. The coupling g_σ and g_ω are chosen to reproduce to the saturation properties of nuclear matter. We remark that the baryonic chemical potential μ_0 at the normal density is equal to $M - a_1 = 923$MeV, where a_1 is the binding energy of a nucleon in nuclear matter.

For the quark phase, we use the two flavor NJL-type model with vector interaction [3] which is described by the following Lagrangian.

$$
\mathcal{L}_{\rm NJL} = \bar{q}(i\gamma^\mu\partial_\mu - m_0)q + \frac{G_{\rm s}}{2}\left((\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right) - \frac{G_{\rm v}}{2}(\bar{q}\gamma^\mu q)^2. \quad (2)
$$

1

The G_s and G_v are the coupling constants for the scalar and the vector-type interactions, respectively. The G_s and the cut-off Λ are chosen to reproduce the pion decay constant f_π and the pion mass m_π, while we treat G_v as a free parameter. [4] If we introduce the auxiliary meson fields

$$\vec{\tilde{\pi}} = \bar{q}i\gamma_5\vec{\tau}q, \quad \tilde{\sigma} = \bar{q}q, \quad \tilde{\omega}_\mu = \bar{q}\gamma_\mu q$$

, which correspond to the meson fields in the Walecka model, $\mathcal{L}_\mathrm{NJL}$ is rewritten as follows. [5]

$$\mathcal{L}_{\mathrm{q}\pi\sigma\omega} \;\; = \;\; \bar{q}(i\gamma^\mu\partial_\mu - m_0)q + G_\mathrm{s}\left(\tilde{\sigma}\bar{q}q + \vec{\tilde{\pi}}\cdot\bar{q}i\gamma_5\vec{\tau}q\right) - G_\mathrm{v}\tilde{\omega}^\mu\bar{q}\gamma_\mu q - \frac{G_\mathrm{s}}{2}\left(\tilde{\sigma}^2 + \tilde{\pi}^2\right) + \frac{G_\mathrm{v}}{2}\tilde{\omega}^\mu\tilde{\omega}_\mu$$

To determine which phase is realized, we should compare pressures in two phases. Because of the saturation properties of nuclear matter, $P_\mathrm{H} = 0$ at the normal density. Therefore, for realization of the hadron phase at the normal density,

$$P_\mathrm{Q}(T = 0, \mu = \mu_0) < P_\mathrm{H}(T = 0, \mu = \mu_0) = 0$$

is needed. Since

$$\left.\frac{\partial P_\mathrm{Q}}{\partial \mu}\right|_{T=0} = \frac{n_\mathrm{Q}}{3} \geq 0,$$

where n_Q is the quark number, the following inequality should be satisfied.

$$-B \equiv P_\mathrm{Q}(T = 0, \mu = 0) \overset{<}{_=} P_\mathrm{Q}(T = 0, \mu = \mu_0) < 0$$

In the following, we call B a "bag constant". Therefore, a positive bag constant is needed for the realization of the hadron phase at the normal density.

3 Numerical results

For numerical calculations, we use the mean field approximation. In Fig. 1, we show the pressures of both phases at $T = 0$ as functions of μ^4. It is seen that the hadron phase is realized at high density, if vector coupling is strong in the quark phase. (In this figure, we put $B = 0$. Realization of the quark phase is less favorable for the case of a positive bag constant.) For the realization of the quark phase at high density, strong vector coupling is needed in the hadron phase, while it is forbidden in the quark phase.

It may be natural that, at low density, the strong vector-coupling in Eq. (1) is related to the vector coupling in Eq. (2). If we add the multi-quark interaction

$$\mathcal{L}_\mathrm{MQI} = -\frac{G_\mathrm{sv}}{2}\left((\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right)(\bar{q}\gamma^\mu q)^2$$

to $\mathcal{L}_\mathrm{NJL}$, the effective vector coupling $G_\mathrm{v}^* = G_\mathrm{v} + G_\mathrm{sv}\tilde{\sigma}^2$ decreases as the density increases. (See Fig. 2.) [4] As is seen in Fig. 1, the quark-phase is realized at high density even if $G_\mathrm{v}' \equiv G_\mathrm{v}^*(\mu = 0)$ is large. With an assumption $G_\mathrm{vH}(\equiv \frac{g_\omega^2}{m_\omega^2}) \sim 3G_\mathrm{v}'$, this result may indicate that the strong vector coupling in the hadron phase is induced by the chiral symmetry breaking.

4 Summary

In summary, we have investigated a role of repulsive force in the quark-hadron phase transition at high density. For realization of the quark phase at high density,

strong vector coupling is needed in the hadron phase, while it is forbidden in the quark phase. The NJL-type model with a multi-quark interaction may relate the weak vector coupling in the quark phase to the strong vector coupling in the hadron phase. This result may indicate that the strong vector coupling in the hadron phase is induced by the chiral symmetry breaking.

References

[1] See, e.g., H. Kouno and F. Takagi, Z. Phys. **C42** (1989) 209 , and references therein.

[2] J. D. Walecka, Ann. Phys. **83** (1974) 491.

[3] See, e.g., M. Kitazawa, T. Koide, T. Kunihiro and Y. Nemoto, Prog. Theor. Phys. **108** (2002) 929, and references therein.

[4] K. Kashiwa, H. Kouno, T. Sakaguchi, M. Matsuzaki and M. Yahiro, arXiv:nucl-th/0608078(SAGA-HE-229-06).

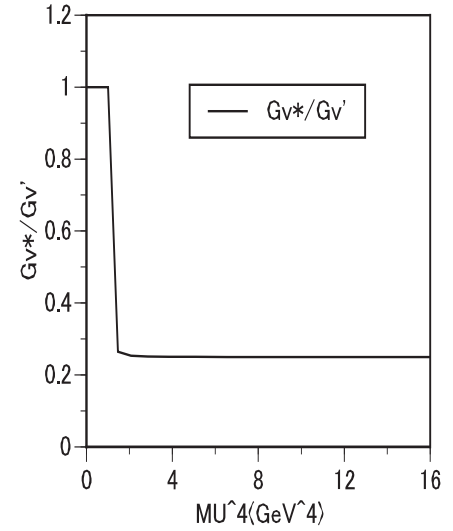[5] See, e.g., T. Sakaguchi, M. Matsuzaki, H. Kouno and M. Yahiro, arXiv:hep-ph/0606219, and references therein.
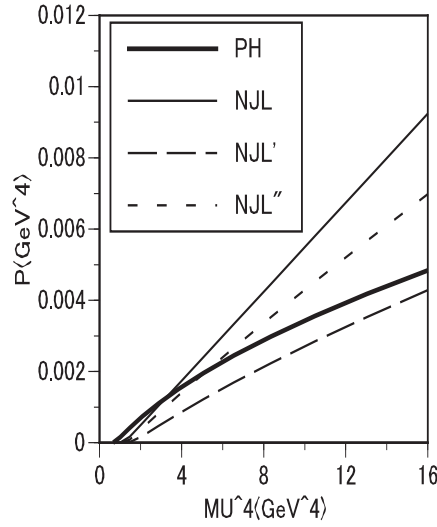
P_{H} : Walecka model.
NJL : $G_{\mathrm{v}} = 0$.
NJL$'$: $G_{\mathrm{v}} = G_{\mathrm{s}}/1.5$.
NJL$''$: $\mathcal{L}_{\mathrm{NJL}} + \mathcal{L}_{\mathrm{MQI}}$. $G_{\mathrm{v}}' = G_{\mathrm{s}}/1.5$.

Fig.1 Pressures in the hadron and the quark phase.   **Fig.2** Effective vector coupling in NJL$''$.